A new Wave Front Sensor for only $\pi/2$ euros !

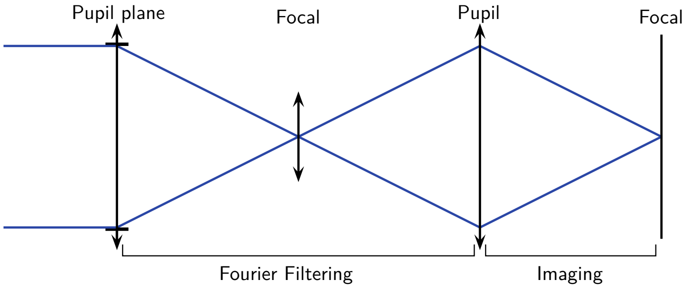
Olivier Fauvarque[φ], **Victoria Hutterer**[Σ], **Yoann Brûlé**

P. Janin-Potiron, J. Duboisset, C. Correia, B. Neichel, V. Chambouleyron,
Iu. Shatokhina, R. Ramlau, J.-F. Sauvage, T. Fusco

AO4ELT6 - June 11, 2019

912 kg CO_2

Coronagraphy & Fourier based Wave Front Sensing

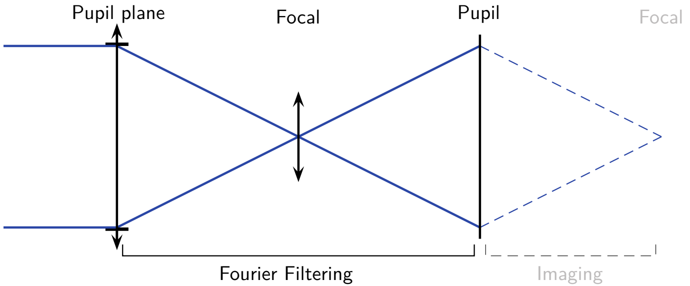


Coronagraphy

Mask **Lyot stop** **Detector**

→ Lyot
→ Roddier&Roddier
→ 4 quadrants phase mask
→ Vortex

The mask has to... **reject light** outside the pupil image.



Wave Front Sensing

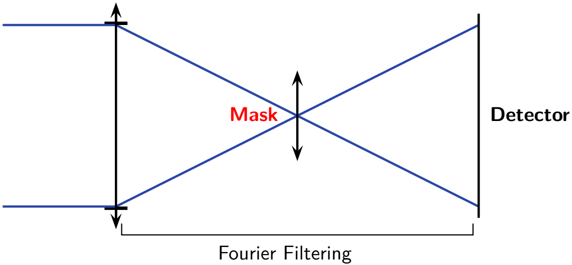
Mask

Detector

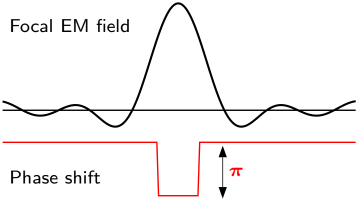
→ Foucault knife edge
→ Zernike mask
→ Pyramids

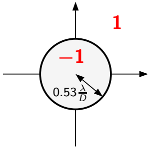
The mask has to... **create a bijection** btw phase and intensity spaces.

Some examples

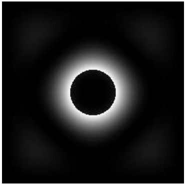


The Roddier&Roddier Coronagraph





R&R mask
transparency function



Energy distribution in
the Lyot's stop plane

Rejection of the light !

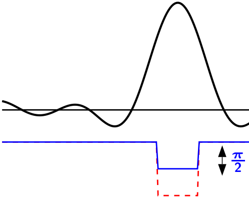
→ **Opposition of phase**

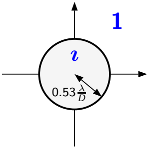
CORONAGRAPH !

→ Destructive interference !

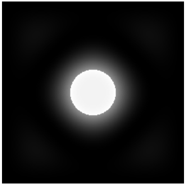
(but... no bijection)

The Zernike Wave Front Sensor





Zernike mask
transparency function



Intensity in the
detector plane

→ **Quadrature of phase**

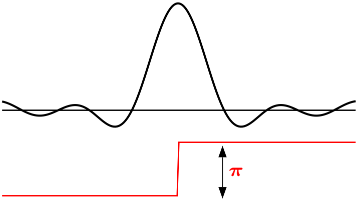
→ Constructive interference !

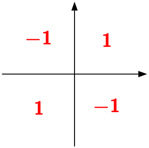
Bijection !

WAVE FRONT SENSOR !

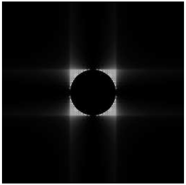
Phase contrast method

The Four Quadrant Phase Mask Coronograph





FQPM mask
transparency function



Energy distribution in
the Lyot's stop plane

Rejection of the light !

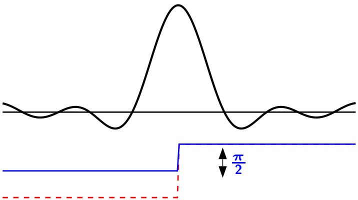
CORONAGRAPH !

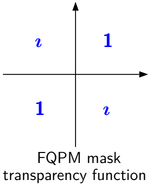
(but... no bijection)

$\rightarrow$ **Opposition of phase**

$\rightarrow$ Destructive interference !

A new wave front sensor ?





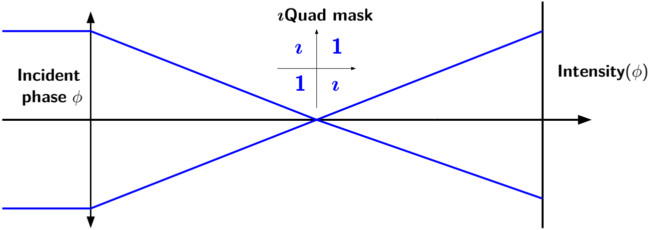
Bijection ?

→ **Quadrature of phase**

WAVE FRONT SENSOR ?

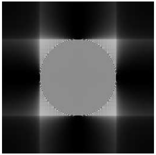
→ Constructive interference !

The $\imath$Quad



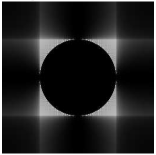
Is it a (good) wave front sensor ?

Reference intensity



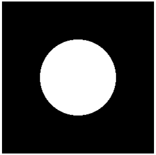
$= \frac{1}{2}$

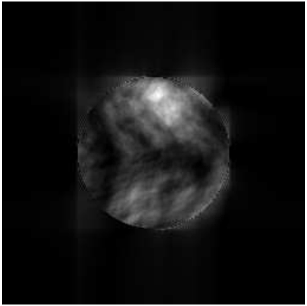
FQPM corona.



$+ \frac{1}{2}$

Entry pupil





Detector intensity

Promising first results !

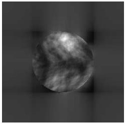
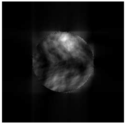
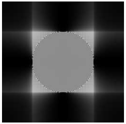
Interesting properties

0. $\imath$Quad output
1. $\imath$Quad & Pyramid
2. Adjoint & model-based phase reconstructors
3. Sensitivity

ıQuad output

Return-to-reference + Flux normalization

Output(ϕ) = **Intensity**(ϕ) - **Intensity**(0)

 =  - 

→ Quantity to **invert** to **measure** the phase.

$$\begin{aligned} \text{Output}(\phi) &= \textbf{Linear behavior}(\phi) + \text{Non-linearities}(\phi) \\ &\approx \textbf{Linear behavior}(\phi) \quad \text{in closed loop} \end{aligned}$$

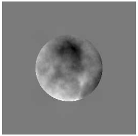
$$\textbf{Linear behavior}(\phi)(x,y) = \frac{\mathbf{I}_P(x,y)}{\pi^2} \iint_P \frac{\phi(x,y) - \phi(x',y')}{(x-x')(y-y')} \mathrm{d}x' \mathrm{d}y'$$

→ **Very simple expression !**

→ Major role played by the **2D Hilbert transform**

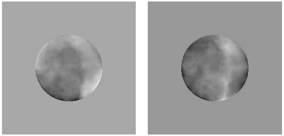
Link with the slopes maps

$$\textbf{Linear Behavior}\Big(\phi\Big)(x,y) = \frac{\mathbf{I}_P(x,y)}{\pi^2}\iint \frac{\phi(x,y)-\phi(x',y')}{{\color{blue}(x-x')}{\color{red}(y-y')}}\mathrm{d}x'\mathrm{d}y'$$



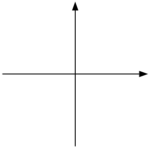
$$\textbf{Slopes}_x\Big(\phi\Big)(x,y) = \frac{\mathbf{I}_P(x,y)}{\pi}\int \frac{\phi(x,y)-\phi(x',y)}{{\color{blue}x-x'}}\mathrm{d}x'$$

$$\textbf{Slopes}_y\Big(\phi\Big)(x,y) = \frac{\mathbf{I}_P(x,y)}{\pi}\int \frac{\phi(x,y)-\phi(x,y')}{{\color{red}y-y'}}\mathrm{d}y'$$



$\imath$Quad output = ***Combination*** of the Pyramid slopes maps !

Why ?
Same splitting
of the focal plane
Cartesian tesselation



$\imath$Quad **adjoint**

$$\left\langle \phi_1 \middle| \textbf{Linear Behavior}(\phi_2) \right\rangle = \left\langle \textbf{Linear Behavior}(\phi_1) \middle| \phi_2 \right\rangle$$

The $\imath$Quad is **self-adjoint !**

→ Very interesting for some **phase reconstruction algorithms** !

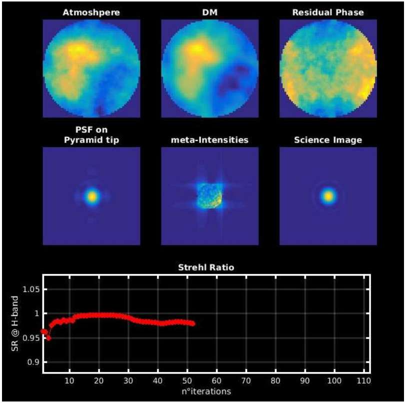
Iterative Landweber algorithm

$$\phi_{k+1} = \phi_k + \alpha \mathbf{Q}^* \left[s - \mathbf{Q}\phi_k \right]$$

Q : Linear Behavior operator
$\mathbf{Q}^*$: adjoint of **Q**
s : signal to invert
α : relaxation parameter
ϕ_k : estimated phase at step k

$\imath$Quad $\mathbf{Q} = \mathbf{Q}^*$

Could we use this algorithm to **close a loop ?**



Simulations done by V. Hutterer on OOMAO

It's working !

Strehl ratio improvment !

and without calibration

But there are some issues :

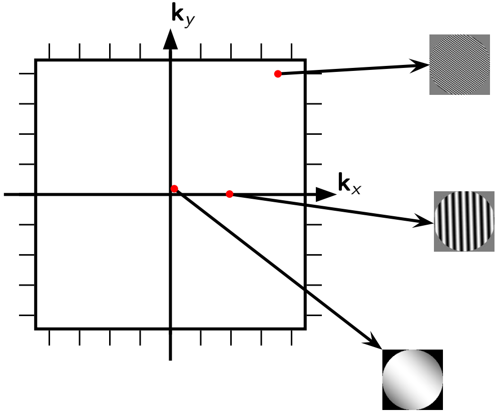
- Only effective if phases to be measured are already small...

→ Second AO stage ?

- Not perfect reconstruction...

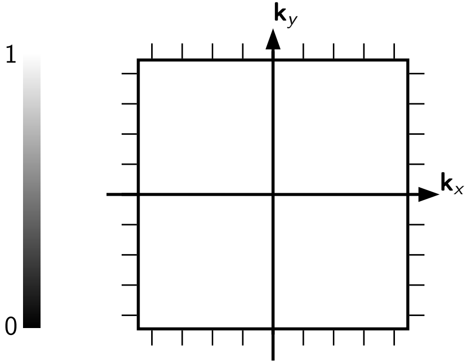
→ Some modes are badly seen...

Sensitivity wrt the **spatial frequencies**

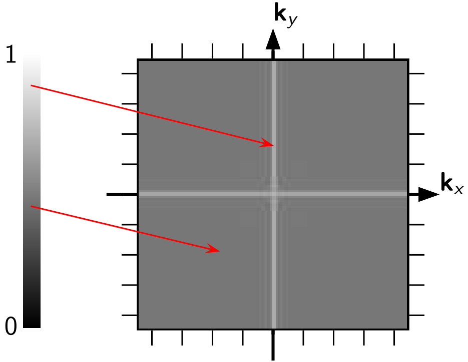


Sensitivity wrt the **spatial frequencies**

Norm of the **Linear Behavior**($\phi_{\mathbf{k}_x,\mathbf{k}_y}$)

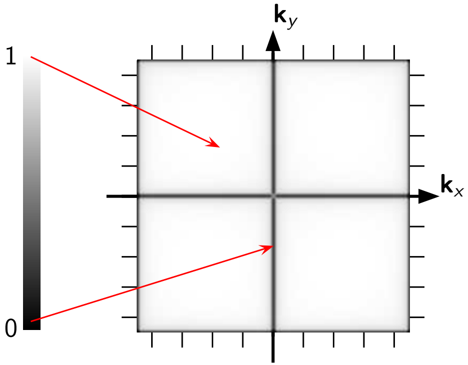


Sensitivity wrt the **spatial frequencies**



Pyramid WFS (without modulation)

Sensitivity wrt the **spatial frequencies**



ıQuad WFS

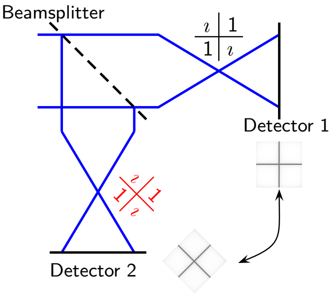
$\imath$Quad sensitivity

- **Mostly better** than the Pyramid WFS...
- ...but **null measurement space**...

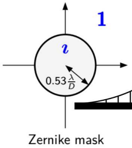
→ Explains why the reconstruction is not perfect !

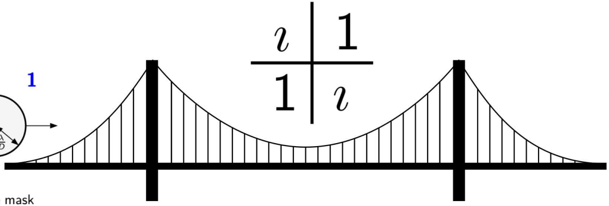
- **Solution ?**

→ **Filling up** the frequencies space with a 2 paths device !



Conclusions & Perspectives





- Comes from coronagraphy
- Phase contrast method

- Cartesian geometry
- Hilbert transforms
- Slopes maps analogy

\+ Simple mathematical description

\+ Self-adjoint → Algorithms !

\- Null measurement space...

→ Two paths solution

Theory	Non-linearities ?
Simulations	Rigorous **comparison** with other WFSs ?
Experiments	**Validation** on the LOOPS bench ?

For which applications ?

Appendix

Propagation operator

...allows to calculate the detector field depending on the incoming EM field.

$$\mathcal{W}_{\text{Pyramid}} = \frac{1}{4}\mathcal{T}_{\alpha,\alpha}\left[\mathcal{I} - \mathcal{H}_{xy} - \imath(\mathcal{H}_x + \mathcal{H}_y)\right] + \frac{1}{4}\mathcal{T}_{-\alpha,\alpha}\left[\mathcal{I} + \mathcal{H}_{xy} + \imath(\mathcal{H}_x - \mathcal{H}_y)\right] + \frac{1}{4}\mathcal{T}_{\alpha,-\alpha}\left[\mathcal{I} + \mathcal{H}_{xy} - \imath(\mathcal{H}_x - \mathcal{H}_y)\right] + \frac{1}{4}\mathcal{T}_{-\alpha,-\alpha}\left[\mathcal{I} - \mathcal{H}_{xy} + \imath(\mathcal{H}_x + \mathcal{H}_y)\right]$$

$$\mathcal{W}_{\imath\text{Quad}} = \frac{\mathcal{I} + \imath\mathcal{H}}{\sqrt{2}}$$

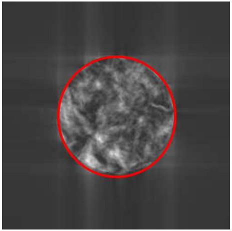
Pyramid mask
Apex angle $= \alpha$

$\mathcal{I}$	Identity Operator
$\mathcal{H}_x$	1D Hilbert transform along x
$\mathcal{H}_y$	1D Hilbert transform along y
$\mathcal{H}_{xy}$	2D Hilbert transform
$\mathcal{T}_{\alpha,\alpha}$	Translation operator

Where is the ϕ-**linear information** ?

$$\textbf{Linear behavior}(\phi)(x,y) = \frac{\mathbf{I}_P(x,y)}{\pi^2} \iint_P \frac{\phi(x,y)-\phi(x',y')}{(x-x')(y-y')}\mathrm{d}x'\mathrm{d}y'$$

→ **Support in the pupil's geometrical image !**



Detector intensity

- Inside : **linear** + non-linear...
- Outside : only non-linearities...

→ ***ı*Quad** : Efficient in terms of detector size !

$=$	Zernike WFS	1 pix/subaperture
$>$	Flattened Pyramid	>1 pix/subaperture
$\gg$	Classical Pyramid	4 pix/subaperture