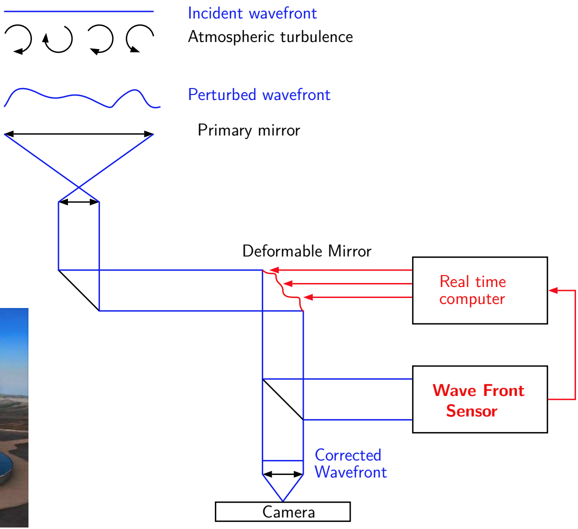
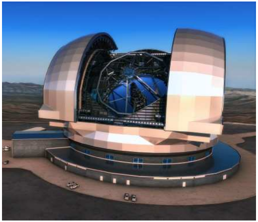
Fourier-based WaveFront Sensing at LAM
A quick overview

Olivier Fauvarque
on behalf of the LAM's Pyramid Team

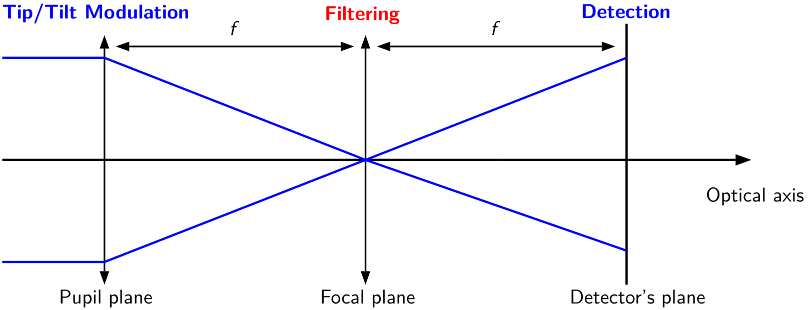
LESIA - HRAA seminar - 20th March 2019

Context: Adaptive Optics for ELTs





What is a **Fourier-based** WFS ?

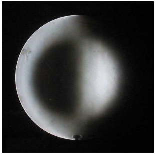


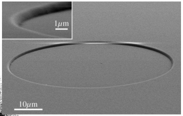
Coherent shaping: **Fourier Filtering**

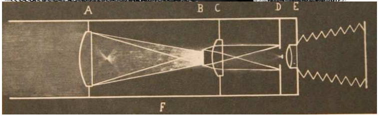
Mask on the focal plane

Incoherent shaping: **Modulation**

Oscillating tip/tilt mirror + Synchronized detector







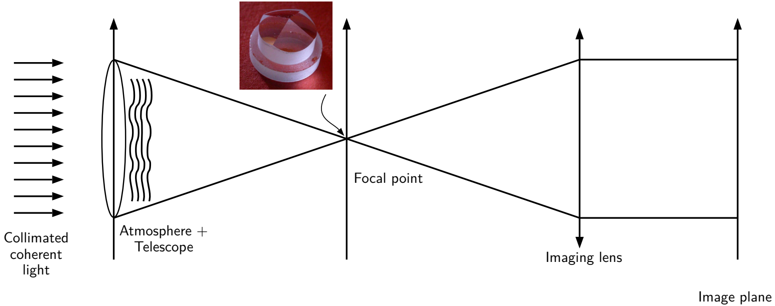
Foucault (1858)
Knife-edge
→ Quality test for mirrors

Zernike (1930s)
Phase contrast method
→ Zernike WFS

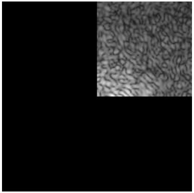
Lyot (1931)
Lyot's pupil in the the stage of its coronagraph

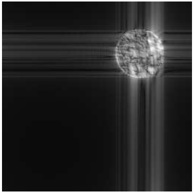
Ragazzoni (1996)
"Smart" all-in-one Knife edge
→ Pyramid WFS

Pyramid Wave Front Sensor

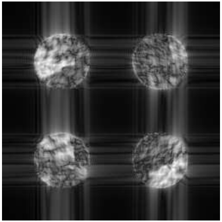



Incident phase

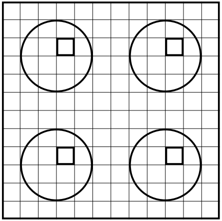

Field in focal plane


Intensity

Pyramid Wave Front Sensor

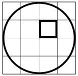


I_1 I_2

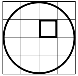


I_4 I_3

Slopes Maps processing



$$S_y = I_1 + I_2 - I_3 - I_4$$

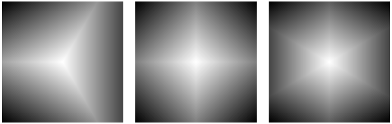


$$S_x = I_2 + I_3 - I_1 - I_4$$

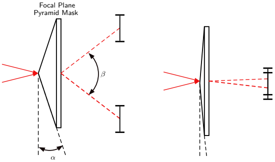
- Numerical processing performed on intensities
- **Incoherent** recombination of phase information !
- Physically: approximately $\partial_x \phi$ & $\partial_y \phi$

Pyramid WFSensor Class

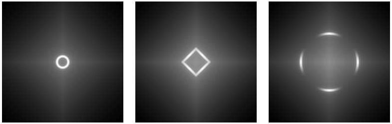
Number of **faces**



Apex **angle**



Weighting function of the modulation
(and Extended object)



The WOLF ANR
Wave-front sensors for adaptive Optics on Extremely Large telescope using Fourier filtering

High contrast Imaging

- LGS WFS

  Sensing with elongated spot ?
- NGS WFS

  Sensitivity and ultimate accuracy

- **Collaborators !**

  LAM LESIA INAF Durham

- Theoretical Model
- Testbeds
- Professional Telescopes

  CANARY + Keck

Part 1: Experimental results

The LOOPS Bench

Playing with the SLM

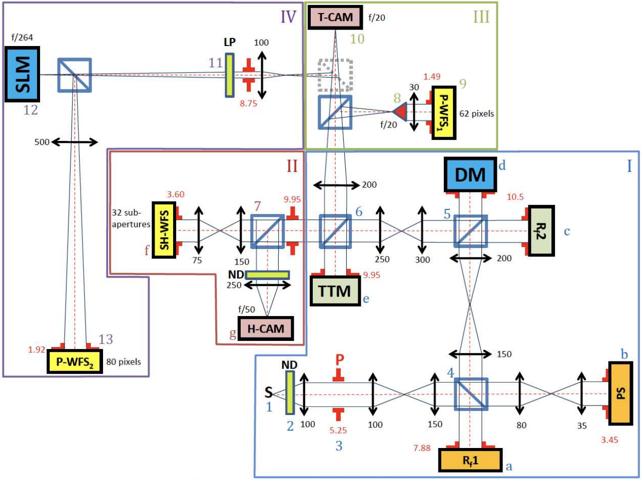
Closing the loop

Part 2: Latest theoretical developments

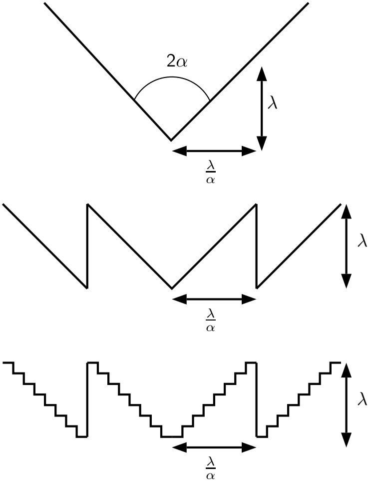
Tackling the **non-linearity**

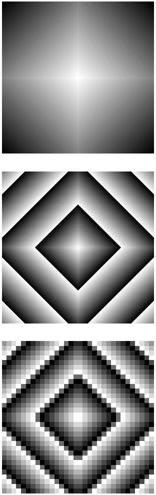
The LOOPS bench

K. El Hadi, C. Bond, Vincent Chambouleyron, Lauren Schatz, **Pierre Janin-Potiron**



The magical Spatial Light Modulator

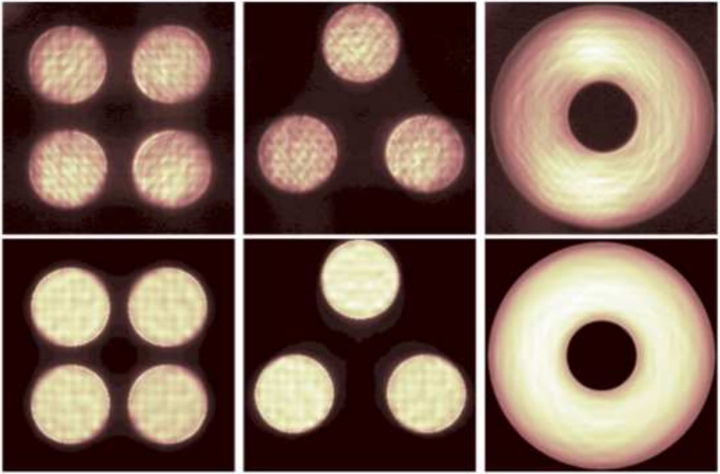




Chromatic, Polarized light but... **extremely flexible** !

Playing with the SLM ! Credit: Lauren Schatz

Exploring the Pyramid WFS class !



Pierre Janin-Potiron et al. Adaptive optics with Fourier-based wavefront sensors: characterization and closed-loop performances on the LOOPS testbed. JATIS March 2019

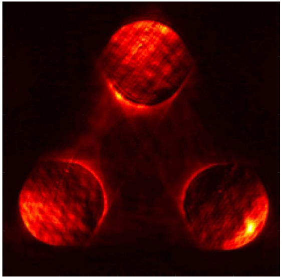
Closing the LOOP: Calibration

Pierre Janin-Potiron   Lauren Schatz   Vincent Chambouleyron

- 4-sided PWFS
- 3-sided PWFS
- Axicon WFS
- Flattened WFS
- Zernike WFS

Closing the loop with a... **3-Sided Modulated Pyramid !**

Lauren Schatz



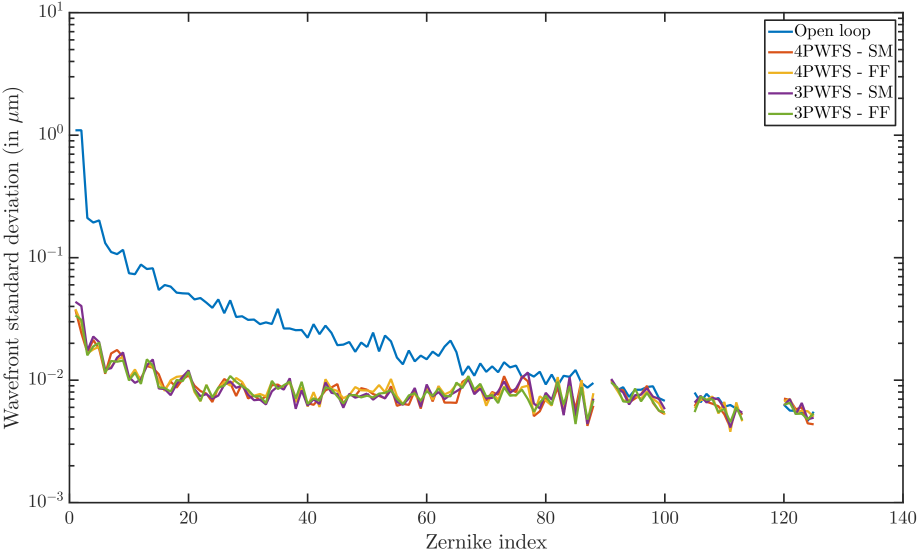
Full frame but also... **slopes maps** for only 3 pupil images !

$$S_x = \frac{I_2\left(\frac{\sqrt{3}}{2}\right) - I_3\left(\frac{\sqrt{3}}{2}\right)}{I_1 + I_2 + I_3}$$

$$S_y = \frac{I_1 - I_2\left(\frac{1}{2}\right) - I_3\left(\frac{1}{2}\right)}{I_1 + I_2 + I_3}$$

Closing the loop... **3 versus 4**-sided Pyramids

Pierre Janin-Potiron, Lauren Schatz



LOOPS testbed... Perspectives

- Rigorous comparison of Fourier-based WFSs:
  - **Low Flux behavior**
  - Able to bootstrap ?
  - Final Strehl ratio ?
  - Shape of the residual PSD ?

  - What are the useful pixels ?
  - **Full frame vs Slopes maps** ?

  - Using the modulation to study WFSensing with extended objects

→ How do we choose the **optimal FbWFS depending on the AO context** ?

Latest theoretical developments

Tackling the **non-linearity**

O. Fauvarque, V. Chambouleyron

What is a **Wave Front Sensor** ?

$$\textbf{Phase} \xrightarrow[\text{Optical propagation}]{\text{WFS}} \text{Intensity} \xrightarrow[\text{Numerical processing}]{\text{Computer}} \textbf{Reconstructed phase}$$

Information: Phase
Information vector: Photon Flux

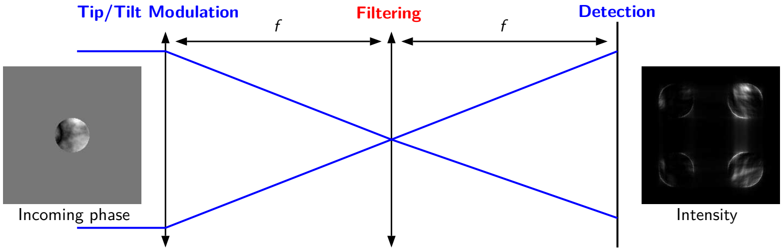
WaveFront Sensor = Converting ϕ-fluctuations into I-fluctuations...

...without ambiguity:

Bijection between:
Phase & intensity

Reconstruction of the phase
$\leftrightarrow$ Inverting $I(\phi)$

Intensity on the detector



Integration during the cycle **Spatial filtering**

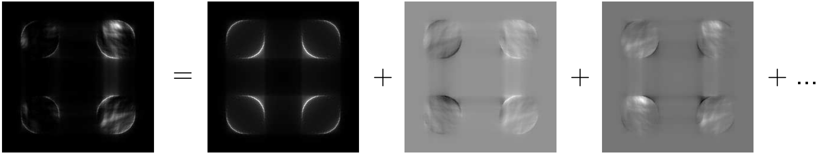
$$I(\phi) = \int_{\mathbb{R}^2} \mathrm{d}a_1 da_2 \; w(a_1, a_2) \left| \; \hat{m} \star \left[\mathbb{I}_P \exp\left(\imath(\phi_m(a_1, a_2) + \phi) \right) \right] \right|^2$$

Modulation

Taylor's development of $I(\phi)$ regarding to ϕ

$$e^{\imath\phi} = 1 + \imath\phi - \frac{\phi^2}{2} + \ldots$$

$$I(\phi) = I_{\text{constant}} + I_{\text{linear}}(\phi) + I_{\text{quadratic}}(\phi) + I_{\text{cubic}}(\phi) + \ldots$$



WFS's output ? $\quad \mathbf{WFS}(\phi) := I(\phi) - I_{\text{constant}}$

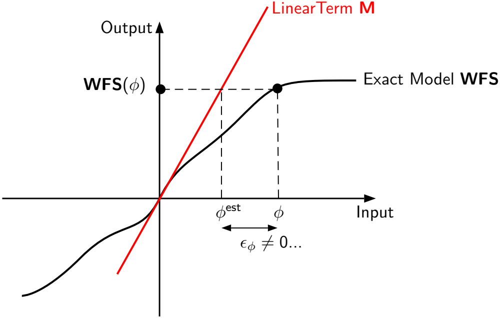
$$= I_{\text{linear}}(\phi) + \textbf{Non-linearities}(\phi)$$

Non-linearities $\rightarrow$ Quadratic + Cubic + Power 4 + etc.

Inverting WFS(ϕ) is not easy at all... unless ...

Non-linearities $<<$ I_{linear}

- Calibration matrix $\mathbf{M} = I_{\text{linear}}(\text{Phase Basis})$
- Phase reconstructor ! Pseudo-inverse $\mathbf{M}^{\dagger} = ({}^t\mathbf{M}\mathbf{M})^{-1}\ {}^t\mathbf{M}$



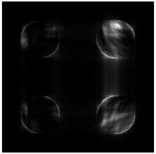
OK... for small phases
BUT... Non-linearities badly affect the reconstruction for higher phases... (Bootstrap, strong residual phases, etc.)

How could we handle the non-linearities ?

1. An **optical** trick:
The **Conjugated Masks** method

Another decomposition

$$I(\phi) = I_{\mathrm{odd}}(\phi) + I_{\mathrm{even}}(\phi)$$



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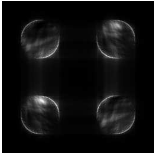

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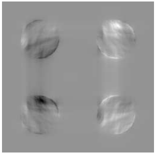


- $I_{\mathrm{even}} = I_{\mathrm{constant}} + I_{\mathrm{quadratic}} + \dots$

  $I_{\mathrm{even}} > 0$

  $I_{\mathrm{even}}(-\phi) = I_{\mathrm{even}}(\phi)$

  Non-bijective...

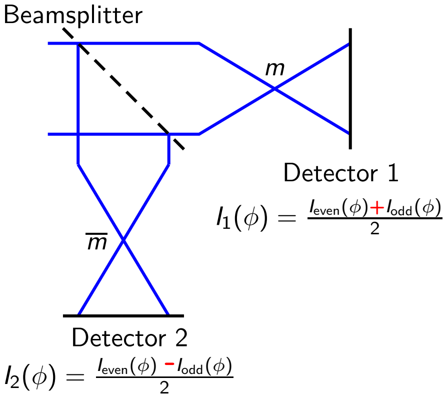
- $I_{\mathrm{odd}} = I_{\mathrm{linear}} + I_{\mathrm{cubic}} + \dots$

  $I_{\mathrm{odd}}(-\phi) = -I_{\mathrm{odd}}(\phi)$

  Bijective !

Would it be possible to kill the even term ?

The **Conjugated Masks** method



$\rightarrow$ The new WFS's output is...

$$\begin{aligned} \textbf{WFS}(\phi) &:= I_1(\phi) - I_2(\phi) \\ &= I_{\text{odd}}(\phi) \end{aligned}$$

$\rightarrow$ Quadratic, Power 4, etc. **BYE BYE !**

$$\textbf{WFS}(\phi) = I_{\text{linear}}(\phi) + I_{\text{cubic}}(\phi) + I_5(\phi) + \ldots$$

WFS's linearity $\nearrow$

2. A **signal processing** approach: Non-linearity seen as **noise**

$$\mathbf{WFS}(\phi) = I_{\text{linear}}(\phi) \quad + \quad \mathbf{Noise}_{\text{non-linear}}(\phi)$$

Adapting the reconstructor to the sensing context !!

Maximum likelihood

$$\mathbf{M}^{\dagger} = ({}^{t}\mathbf{M}\mathbf{C}_{\mathrm{nl}}^{-1}\mathbf{M})^{-1}\ {}^{t}\mathbf{M}\mathbf{C}_{\mathrm{nl}}^{-1}$$

where $\mathbf{C}_{\mathrm{nl}}$ is the **non-linearity noise** **covariance matrix** !

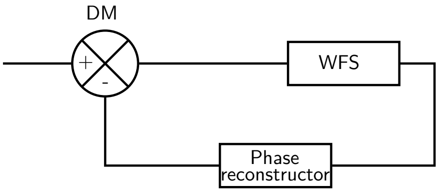
Tricky calculation... which requires **strong assumptions** but...

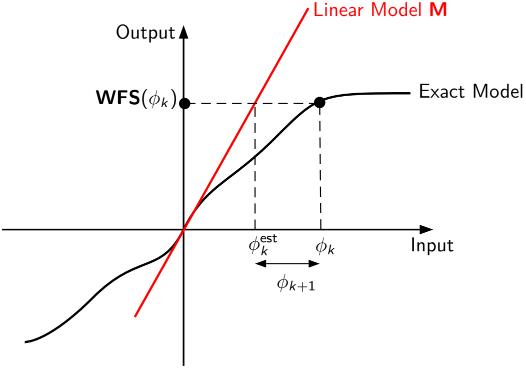
Analytical expression of $\mathbf{C}_{\mathrm{nl}}(\mathbf{m}, \mathbf{w}, \mathbf{B}_{\phi})$

Spatial phase covariance matrix: $\mathbf{B}_{\phi}(\mathbf{r}) = <\phi(\mathbf{r'})\phi^{*}(\mathbf{r'} + \mathbf{r})>$

On going work... First results in June at AO4ELT conference

3. An **algorithmic** approach:





AO loop $\leftrightarrow$ ***Linear* Iterative Algorithm**

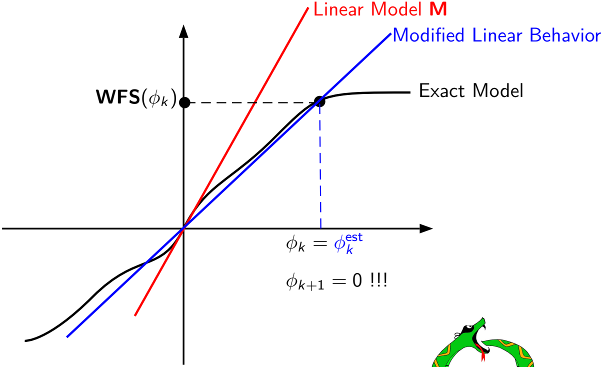
$$\phi_{k+1} = \phi_k - \mathbf{M}^\dagger \left[\mathbf{WFS}(\phi_k)\right]$$

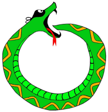
Why **"Linear"** ?
Because $\mathbf{M}^\dagger$ is constant regarding to k.

Non-linear iterative algorithm

"Non-linearity will affect the linear behavior of the sensor."
$\rightarrow$ Adapting the reconstructor to the phase-to-be measured

$$\phi_{k+1} = \phi_k - \mathbf{M}^{\dagger}_{\phi_k}\left[\mathbf{WFS}(\phi_k)\right]$$

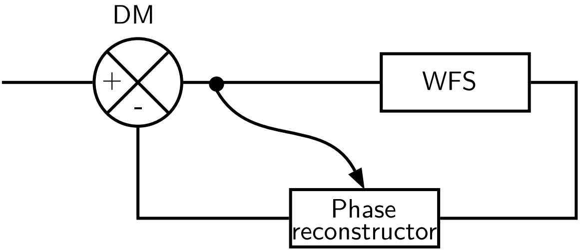




But it is weird... To go from $\mathbf{M}^{\dagger}$ to $\mathbf{M}^{\dagger}_{\phi_k}$
we have to know ϕ_k...
which is the phase-to-be-measured...

A loop in the loop !

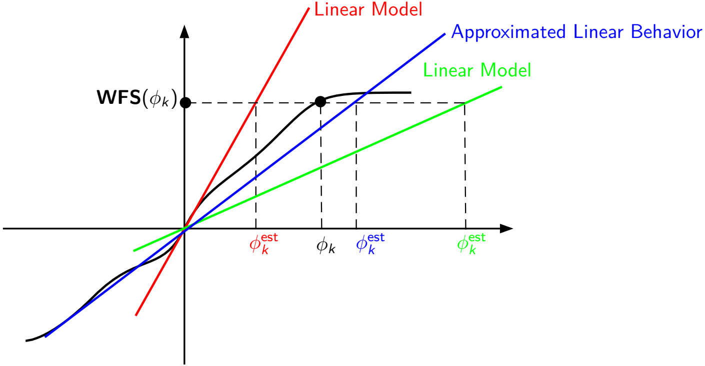
Could we find a **additional signal** which
1) depends on ϕ_k
2) allows to adjust the reconstructor ?



What could be really useful: **WFS'**(ϕ_k)

$\rightarrow$ Linear behavior (or Calibration matrix) around ϕ_k

The reason why knowing **WFS'**(ϕ_k) is useful.



Next step: **Approximating** the Modified Linear Behavior thanks to
- the linear behavior around 0 &
- the linear behavior around ϕ_k

It sounds tricky but actually, it is not a big deal...

A question remains...

What is the **additional signal** allowing to know **WFS'**(ϕ_k)?

We need help from theory...

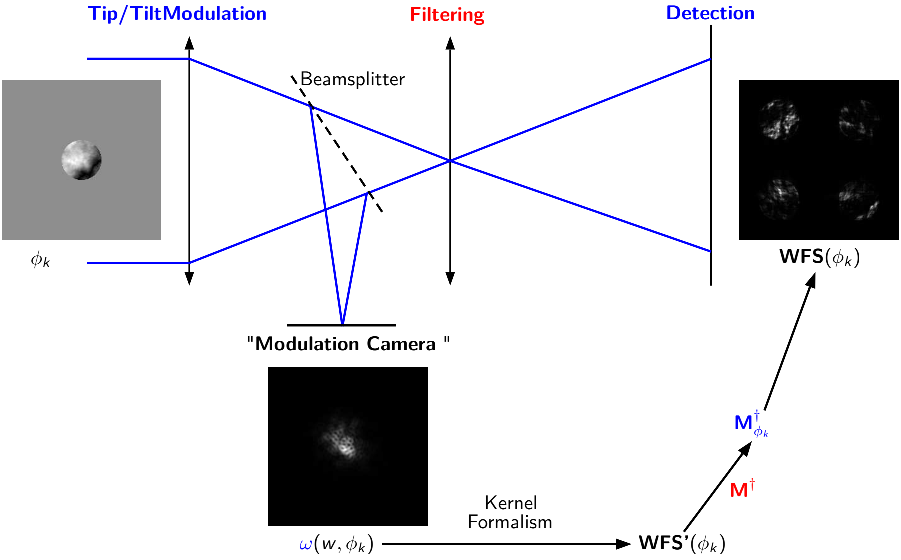
Kernel formalism applied to Fourier based Wave Front Sensing in presence of residual phases

OLIVIER FAUVARQUE,[1,2,*] PIERRE JANIN-POTIRON,[2,3] CARLOS CORREIA,[2] YOANN BRÛLÉ,[2], BENOIT NEICHEL,[2] VINCENT CHAMBOULEYRON,[2] JEAN-FRANCOIS SAUVAGE,[2,3] AND THIERRY FUSCO[2,3]

- Sensitivity of FbWFSs depending on (**m**, **w**, **Operative phase**)
- Sensitivity $\sim$ **WFS'** **Operating phase** $\sim \phi_k$
- Major result: ($\mathbf{m}$,$\mathbf{w}$,ϕ_k) $\rightarrow$ $\left(\mathbf{m},\omega(\mathbf{w},\phi_k)\right)$

Conclusion: To know **WFS'**(ϕ_k) we need the "effective modulation" around ϕ_k

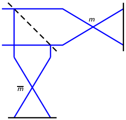
⚠ Not exact... strong assumptions... **better** reconstructor but **not perfect** !



Tip/TiltModulation
Filtering
Detection
Beamsplitter
ϕ_k
$\mathbf{WFS}(\phi_k)$
"Modulation Camera "
$\mathbf{M}^{\dagger}_{\phi_k}$
$\mathbf{M}^{\dagger}$
Kernel
Formalism
$\omega(w, \phi_k)$
$\mathbf{WFS'}(\phi_k)$

Tackling the non-linearity... **conclusions**

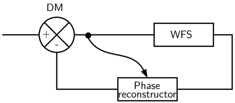
Conjugated masks



Non-linearity noise

$$\mathbf{WFS} = \color{red}{\mathbf{\mathit{I}_{linear}}} + \color{blue}{\mathbf{Noise}}_{\text{non-linear}}$$

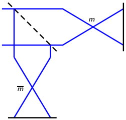
A Loop in the Loop



- **Two outputs**: Pupil & Focal

Tackling the non-linearity... **conclusions**

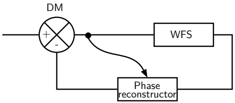
Conjugated masks



Non-linearity noise

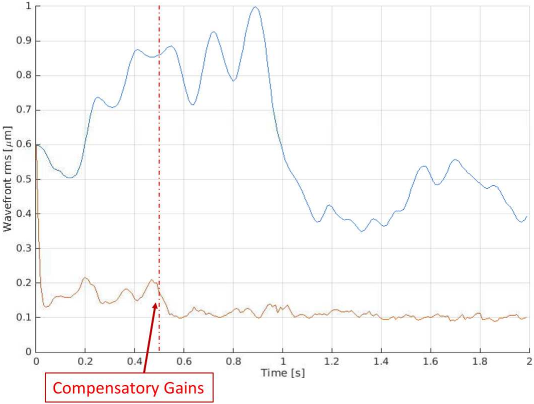
WFS $=$ I_{linear} $+$ **Noise**$_{\text{non-linear}}$

A Loop in the Loop



- **Two outputs**: Pupil & Focal

An application: **Optical Gain Tracking**



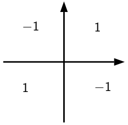
Thank you for your attention

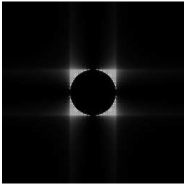
The WOLF ANR
Wave-front sensors for adaptive Optics on Extremely Large telescope using Fourier filtering

APPENDIX

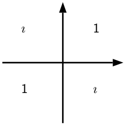
A new WFS coming from the coronagraph world !

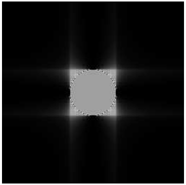
FQPM
Phase shift: π





$\imath$-quad WFS
Phase shift: $\pi/2$





O. Fauvarque, V. Hutterer, **Y. Brûlé** et al. The $\imath$Quad sensor: a new Fourier-based WFS derived from the 4Q-coronagraph Talk (or poster) at AO4ELT conference

Closing the loop...**without using interaction matrix !**

Victoria Hutterer (& Austrian AO team at Linz)   Pierre Janin-Potiron

"Model-based reconstructors"

4-sided modulated pyramid
+ Slopes maps

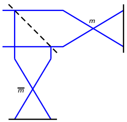
The theory says that:
They are not far from
Shack-Hartmann signal !

→ Using dedicated algorithms to
reconstruct phase.

CuReD & P-CuReD !

Tackling the non-linearity... **conclusions**

Conjugated masks



- **Optical** trick
- $+$ Symmetry=Elegance
- $+$ Bijectivity $\nearrow$
- $+$ Linearity range $\nearrow$

- \- From 1 to 2 paths
- \- ReadOut Noise $\times$ 2

Non-linearity noise

$$\textbf{WFS} = \color{red}{I_{\text{linear}}} + \color{blue}{\textbf{Noise}}_{\text{non-linear}}$$

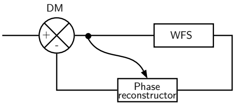
- **Software** approach
- $+$ 1 path only
- $+$ Non-invasive method

- $\pm$ Statistical approach
- $+$ Telemetry
- \- Computing cost: $\mathbf{C}_{nl}$

- \- Strong assumptions

  Will it work ?

A Loop in the Loop



- AO loop = algorithm
- **Two outputs**:

  Pupil & Focal
- $+$ Loop update at each step
- $+$ Extremely robust

  NCPA, Res. phases

- \- Need to **steal photons**

  Trade-off...